A SPACE-TIME ADAPTIVE SCHEME FOR PARABOLIC EQUATIONS

EDUARDO M. GARAU, FERNANDO D. GASPOZ, PEDRO MORIN, AND RAFAEL VÁZQUEZ

Abstract. We present an adaptive algorithm for solving linear parabolic equations using hierarchical splines and the implicit Euler method for the spatial and time discretizations, respectively. Our development follows closely the lines in [4, 3], where fully discrete adaptive schemes have been analyzed within the framework of classical finite elements.

Our approach is based on an a posteriori error estimation that essentially consists of four indicators. On the one hand, we have a time error indicator and a consistency error indicator that dictate the time-step size adaptation. On the other hand, we have a coarsening error indicator and a space error indicator that are used to obtain a suitably adapted hierarchical mesh (at each fixed discrete time). In particular, the considered space error indicators are the function-based a posteriori error estimators for elliptic problems introduced in [1]. The algorithm is guaranteed to reach the final time within a finite number of operations, and keeps the the space-time error below a prescribed tolerance.

We finally present some numerical tests to illustrate the performance of the proposed adaptive algorithm, using the implementation of hierarchical splines presented in [2].

Keywords: A posteriori error estimators, adaptivity, hierarchical splines, parabolic equations

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Universidad Nacional del Litoral and Consejo Nacional de Investigaciones Científicas y Técnicas; Santa Fe; Argentina
E-mail address: egarau@santafe-conicet.gov.ar

Universitt Stuttgart; Stuttgart; Deutschland
E-mail address: fernando.gaspoz@ians.uni-stuttgart.de

Universidad Nacional del Litoral and Consejo Nacional de Investigaciones Científicas y Técnicas; Santa Fe; Argentina
E-mail address: pmorin@santafe-conicet.gov.ar

École Polytechnique Fédérale de Lausanne; Lausanne; Switzerland; and Istituto di Matematica Applicata e Tecnologie Informatiche E. Magenes (CNR); Pavia; Italy
E-mail address: rafael.vazquez@epfl.ch