

# RATE OPTIMAL ADAPTIVE FEM WITH INEXACT SOLVER FOR NONLINEAR OPERATORS

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**ABSTRACT.** We aim to present our recent work [3], where we prove convergence with optimal algebraic rates for an adaptive finite element method for nonlinear equations with strongly monotone operator. We consider an algorithm proposed by [2]. Unlike prior works [4, 1], our analysis also includes the iterative and inexact solution of the arising nonlinear systems by means of the Picard iteration. Using nested iteration, we prove, in particular, that the number of of Picard iterations is uniformly bounded in generic cases, and the overall computational cost is (almost) optimal. Numerical experiments confirm the theoretical results.

**Keywords:** quasilinear elliptic PDE, strongly monotone operators, finite element method, adaptive mesh-refinement, optimal convergence rates.

**Mathematics Subject Classifications (2010):** 65N30, 65N12, 65N50, 65M22, 65J15.

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