OPTIMAL ADAPTIVITY AND PRECONDITIONING FOR THE FRACTIONAL LAPLACIAN

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We present novel inverse estimates for the (integral) fractional Laplacian. More precisely, we show that a weighted L^2-norm, where the weight is a power of the local mesh-width, of the fractional Laplacian can be bounded by the energy norm. Generalizing the arguments used in the boundary element method, [1], the non-local operator is split into a localized near-field and a smoother far-field part, which is treated using the so-called Caffarelli-Silvestre extension problem and interior regularity estimates.

Weighted L^2-norms appear naturally in the context of *a-posteriori* error estimation in adaptive methods. With the help of our inverse estimate, we prove optimal convergence rates of an adaptive algorithm steered by a weighted residual error estimator using the axiomatic approach of [2]. Moreover, we propose a different, reliable weighted error estimator to cover the open case of fractional powers larger than 3/4.

As a second application of the inverse inequalities, we prove that an additive Schwarz preconditioner of BPX-type for the fractional Laplacian on locally refined meshes leads to condition numbers that are uniformly bounded in the refinement level.

References

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