WAVE DIFFRACTION BY RANDOM SURFACES: UNCERTAINTY QUANTIFICATION VIA SPARSE TENSOR BOUNDARY ELEMENTS AND SHAPE CALCULUS

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Abstract. We consider the numerical solution of time-harmonic scattering of acoustic waves from obstacles with uncertain geometries, assuming a small-amplitude random perturbation of a known nominal domain. Using first-order shape derivatives, we derive deterministic boundary integral equations for the mean field and the two-point correlation function of the random solution for all soft-obstacle, hard-obstacle and transmission problems. Sparse tensor Galerkin discretizations of these equations are implemented with the so-called *combination technique*. Consistent discretization errors for the covariance is achieved with $\mathcal{O}(N \log N)$ degrees of freedom instead of $\mathcal{O}(N^2)$. Performance comparison of our approach to classic Monte-Carlo Galerkin formulation is given for different shapes along with a discussion regarding preconditioning and efficient implementation of the scheme. Finally, we verify the robustness of the sparse tensor approximation and compare it to low-rank approximations techniques.

Keywords: uncertainty quantification, sparse tensor approximation, boundary integral equations

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