

# HYBRID HIGH-ORDER METHODS FOR SIGNORINI'S UNILATERAL CONTACT PROBLEM

KAROL CASCAVITA, FRANZ CHOULY, AND ALEXANDRE ERN

ABSTRACT. We devise and analyze a Hybrid High-Order (HHO) method combined with Nitsche's method for scalar-valued diffusion problems equipped with Signorini's unilateral contact conditions. On the one hand, HHO methods have been introduced in [6] for linear diffusion and in [5] for linear elasticity, and these methods have been bridged in [4] to Hybridizable Discontinuous Galerkin methods and nonconforming Virtual Element methods. On the other hand, Nitsche's method [7] is a well-known boundary-penalty technique to enforce weakly Dirichlet boundary conditions. This technique has been recently extended to enforce weakly Signorini's unilateral contact conditions discretized with conforming finite elements in [2, 3]. In the present work, we extend these ideas to the setting of HHO methods. As a first step, we show how to enforce weakly Dirichlet boundary conditions using Nitsche's technique in the context of HHO methods. Therein, the consistency and penalty terms can be written using either the face unknowns or the trace of the cell unknowns, provided the latter are of one degree higher and the local HHO reconstruction operator is slightly modified on the cells having a boundary face. These ideas are inspired from the recent work of [1] on fictitious domain and unfitted HHO methods. Finally, we show that the idea of using the trace of the cell unknowns also yields, in the context of Signorini's unilateral contact conditions, optimal energy-error estimates.

**Keywords:** Hybrid High-Order methods, Signorini's problem, Nitsche's technique

**Mathematics Subject Classifications (2010):** 65N30, 74G15, 74M15

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ENPC, UNIVERSITY PARIS-EST AND INRIA, FRANCE  
*E-mail address:* karol.cascavita@enpc.fr

UNIVERSITÉ DE BOURGOGNE, FRANCE  
*E-mail address:* franz.chouly@u-bourgogne.fr

ENPC, UNIVERSITY PARIS-EST AND INRIA, FRANCE  
*E-mail address:* alexandre.ern@enpc.fr