HDG FOR TRANSIENT ELASTIC WAVES

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Abstract. The Hybridizable Discontinuous Galerkin (HDG) methods are currently in rapid development since they were first proposed by Cockburn et al. [3]. One advantage of the HDG methods is their superconvergence, which can be utilized to obtain post-processed solutions converging faster than the original.

It is known that the superconvergence is lost in elasticity, but there are two major approaches to recover it. The first approach is by M-decomposition [2, 1], in which case the approximation space for stress is enriched by adding more basis functions. However, the added basis can be rational functions instead of polynomials and therefore can lead to some difficulties in implementation. The second approach is by using Lehrenfeld-Schöberl type HDG (LS-HDG) methods [5, 6]. The LS-HDG methods use only polynomial basis functions, achieve superconvergence without post-processing, and their computational complexity is almost the same to the traditional HDG methods. However, the existing error analyses for LS-HDG are all based on orthogonal projections, which make the analysis complicated and detached from the existing projection-based error analysis of HDG methods [4], where tailored projections are used to make the analysis simple and concise.

We present some new techniques that allow a unified framework for M-decomposition and LS-HDG methods. We will show how a tailored projection for the LS-HDG method of linear elasticity can be devised based on these new techniques. As applications, we present a HDG method for elastic waves that has a uniform-in-time superconvergent property. We will show that the proof for superconvergence can be easily obtained by using the newly devised tailored projection and some existing techniques in traditional HDG methods.

Keywords: HDG methods, elastic waves, tailored projection

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