AN ADAPTIVE *hp*–REFINEMENT STRATEGY WITH INEXACT SOLVERS AND COMPUTABLE GUARANTEED BOUND ON THE ERROR REDUCTION FACTOR

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Abstract. We propose a new practical adaptive refinement strategy [1] for *hp*-finite element approximations of elliptic problems. Following recent theoretical developments in polynomial-degree-robust a posteriori error analysis, we solve two types of discrete local problems on vertex-based patches. The first type involves the solution on each patch of a mixed finite element problem with homogeneous Neumann boundary conditions, which leads to an $\mathbf{H}(\mathrm{div},\Omega)$-conforming equilibrated flux. This, in turn, yields a guaranteed upper bound on the error and serves to mark mesh vertices for refinement via Dörfler's bulk-chasing criterion. The second type of local problems involves the solution, on patches associated with marked vertices only, of two separate primal finite element problems with homogeneous Dirichlet boundary conditions, which serve to decide between *h*-, *p*-, or *hp*-refinement. Altogether, we show that these ingredients lead to a computable guaranteed bound on the ratio of the errors between successive refinements (error reduction factor). Under some additional assumptions on the employed refinemnt methods, we are able to prove that the resulting bound on the reduction factor is indeed strictly lower than one. An extension of the proposed strategy [2] by taking into account an inexact algebraic solver is also considered. In a series of numerical experiments featuring smooth and singular solutions, we investigate the practicality of the proposed adaptive solver and show that exponential convergence rates are achieved even in the presence of an inexact solver. We also investigate the accuracy of our bound on the reduction factor by evaluating the ratio of the predicted reduction factor relative to the true error reduction, and we find that this ratio is in general quite close to the optimal value of one.

Keywords: elliptic problem, finite element method, a posteriori error estimate, equilibrated flux, *hp*-adaptivity, error reduction, algebraic error

Mathematics Subject Classifications (2010): 65N30, 65N15, 65N50

References

[1] Patrik Daniel, Alexandre Ern, Iain Smears, and Martin Vohralík. An adaptive *hp*-refinement strategy with computable guaranteed bound on the error reduction factor. *Computers and Mathematics with Applications*, 76(5):967–983, 2018. ISSN 0898-1221.

[2] Patrik Daniel, Alexandre Ern, Iain Smears, and Martin Vohralík. An adaptive *hp*-refinement strategy with inexact solvers and computable guaranteed bound on the error reduction factor. *In Preparation.*

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