NONLOCAL MODELS WITH NONSTANDARD INTERACTION DOMAINS: COMPARATIVE ANALYSIS AND EFFICIENT FINITE ELEMENT METHODS

MARTA D'ELIA, MAX GUNZBURGER, VOLKER SCHULZ, AND CHRISTIAN VOLLMAN

Abstract. Fractional models and more general nonlocal models with finite-range interactions recently gained popularity in several diverse scientific and engineering applications that range from continuum mechanics to stochastic processes. We are particularly interested in nonlocal diffusion operators, i.e. the nonlocal counterpart of elliptic operators for partial differential equations (PDEs) that can be used to describe a large class of applications including nonlocal elasticity, subsurface flow, image processing and nonlocal heat conduction. These models can capture features of the solution that cannot be described by classical PDEs. However, their accuracy comes at a price: the discretization of nonlocal models usually involves dense or full matrices that require a lot of memory storage and whose assembling can be prohibitively expensive. Standard nonlocal models with finite-range interactions use euclidean (or 2-norm) balls as interaction regions. In this work we consider a novel concept of neighborhood where the interaction regions are more general so-called nonstandard interaction sets including, e.g., norm balls which are induced by the infinity- or 1-norm. Initially motivated by computational challenges, this approach can be considered an applicable model in its own right. We present an analysis of the well-posedness of the nonlocal model induced by nonstandard interaction regions and a careful comparison of models induced by different norms. Also, we illustrate our theoretical results with several two-dimensional numerical tests and present an application to image processing, where the use of infinity-balls is induced by the nature of the problem.

Keywords: nonlocal diffusion, nonlocal vector calculus, nonlocal neighborhoods, finite element, image processing.

Sandia National Laboratories, NM
E-mail address: mdelia@sandia.gov

Florida State University
E-mail address: mgunzburger@fsu.edu

University of Trier
E-mail address: volker.schulz@uni-trier.de

University of Trier
E-mail address: vollmann@uni-trier.de