NUMERICAL APPROXIMATION OF THE SPECTRAL FRACTIONAL LAPLACIAN ON BOUNDED DOMAINS

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Abstract. The crucial issue in using differential operators of fractional order for real world applications is that they are naturally defined on the entire space $\mathbf{R}^n$, $n \geq 1$. However, in the majority of practical cases one needs to model quantities that are defined only on a bounded domain $\Omega \subset \mathbf{R}^n$. The main challenge is hence to suitably restrict, adapt, or interpret the definition of a fractional operator so that it preserves its nonlocal character while allowing for a well-posed formulation of the problem on Ω.

We consider a family of operators which are boundary conditions dependent and present a numerical discretization that combines quadratures rules with finite element methods. Relying on the integral formulation of the operator via the heat-semigroup formalism, such formulation can handle at once cartesian domains and multi-dimensional (possibly irregular) geometries [1]. We also discuss the numerical approximation of the corresponding fractional Poisson problem [2], and briefly outline a practical application in cardiac electrophysiology [3].

Keywords: fractional Laplacian, bounded domain, heat-semigroup formulation, finite elements, integral quadrature

Mathematics Subject Classifications (2010): 47G20, 65N30, 41A55, 65N15, 35R11, 26A33, 65R10, 65N25

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