

A HIGHER-ORDER METHOD ON A FITTED MESH FOR TIME-FRACTIONAL INITIAL-VALUE AND INITIAL-BOUNDARY VALUE PROBLEMS

HU CHEN AND MARTIN STYNES

ABSTRACT. Alikhanov's high-order scheme for Caputo fractional derivatives of order $\alpha \in (0, 1)$ is generalised to nonuniform meshes and analysed for initial-value problems (IVPs) and initial-boundary value problems (IBVPs) whose solutions display a typical weak singularity at the initial time. It is shown that, when the mesh is chosen suitably, the scheme attains order $3 - \alpha$ convergence in time for the 1-dimensional IVP and second-order convergence in time for the IBVP. For the IBVP we consider the case where the spatial domain is the unit square and use a spectral method to discretise in space, but other spatial domains and other spatial dimensions and discretisations are possible. Numerical results demonstrate the sharpness of the theoretical convergence estimates.

An improved discretisation for the IBVP attains order $3 - \alpha$ convergence (the same order as the IVP) in numerical experiments, but its analysis remains open.

Keywords: Caputo derivative, Alikhanov scheme, fitted mesh

Mathematics Subject Classifications (2010): 65M06, 65M12, 65M60, 65M70

REFERENCES

- [1] Hu Chen and Martin Stynes. Error analysis of a second-order method on fitted meshes for a time-fractional diffusion problem. *J. Sci. Comput.* (to appear).
- [2] Hu Chen and Martin Stynes. A high order method on graded meshes for a time-fractional diffusion problem. In *Finite Difference Methods, Theory and Applications* (Proc. of Seventh Conference on Numerical Analysis and Applications, Lozenetz, Bulgaria, 11–16 June 2018), Lecture Notes in Comput. Sci. Vol. ???, I. Dimov, I. Faragó & L. Vulkov (Eds.), Springer, Cham, Switzerland, pp.13–24 (to appear).

BELJING COMPUTATIONAL SCIENCE RESEARCH CENTER, BEIJING 100193, CHINA
E-mail address: `huchen90@csrc.ac.cn`

BELJING COMPUTATIONAL SCIENCE RESEARCH CENTER, BEIJING 100193, CHINA
E-mail address: `m.stynes@csrc.ac.cn`