A FULLY-MIXED FINITE ELEMENT METHOD FOR THE COUPLING OF THE NAVIER–STOKES AND DARCY–FORCHHEIMER EQUATIONS

SERGIO CAUCAO, GABRIEL N. GATICA, AND FELIPE SANDOVAL

Abstract. In this work we present and analyse a fully-mixed formulation for the nonlinear model given by the coupling of the Navier–Stokes and Darcy–Forchheimer equations with the Beavers–Joseph–Saffman condition on the interface. Our approach yields non-Hilbertian normed spaces and a twofold saddle point structure for the corresponding operator equation. Furthermore, since the convective term in the Navier–Stokes equation forces the velocity to live in a smaller space than usual, we augment the variational formulation with suitable Galerkin type terms. The resulting augmented scheme is then written equivalently as a fixed point equation, so that the well-known Schauder and Banach theorems, combined with classical results on nonlinear monotone operators, are applied to prove the unique solvability of the continuous and discrete systems. In particular, given an integer $k \geq 0$, Raviart–Thomas spaces of order k, continuous piecewise polynomials of degree $\leq k+1$ and piecewise polynomials of degree $\leq k$ are employed in the fluid for approximating the pseudostress tensor, velocity and vorticity, respectively, whereas Raviart–Thomas spaces of order k and piecewise polynomials of degree $\leq k$ for the velocity and pressure, constitute a feasible choice in the porous medium. *A priori* error estimates and associated rates of convergence are derived, and several numerical examples illustrating the good performance of the method are reported.

Keywords: Navier–Stokes equation, Darcy–Forchheimer equation, twofold saddle point, fixed point theory, augmented fully-mixed formulation, mixed finite element methods.

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Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción, Casilla 160-C, Concepción, Chile

E-mail address: scaucao@ci2ma.udec.cl

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile

E-mail address: ggatica@ci2ma.udec.cl

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Casilla 160-C, Concepción, Chile

E-mail address: fsandovals@udec.cl