NUMERICAL SIMULATIONS OF SEDIMENTATION IN AXISYMMETRIC VESSELS

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Abstract. The sedimentation of an ideal suspension in axisymmetric vessels can be modeled by a coupled system of conservation equations in cylindrical coordinates, and under the assumption of rotationally symmetric solutions, the spatial coordinates are the radius and height. The approach presented is based on the local solids volume fraction, the mass-average velocity and the excess pore pressure in a similar way as [3]. The full coupled system is composed by a scalar advection equation for the local solids volume fraction and a stationary Stokes system with non-zero divergence for the velocity and pressure. The equation for the divergence of the velocity field has an extra term that depends on the derivatives of the pressure. This term enforces extra regularity on the pressure. Furthermore, the local solids volume fraction exhibits discontinuities which imposes difficulties in the numerical approximation.

A numerical scheme for the approximation of the solution is presented. The method is based on the discontinuous Galerkin method for the advection equation and a stabilized continuous Galerkin method with $\mathbb{P}_1$-$\mathbb{P}_1$ elements for the Stokes system, see [1]. Numerical simulations for different geometries are also presented.

This work has partly been inspired by [2].

Keywords: axisymmetric formulation, sedimentation process, clarifier-thickener units

Mathematics Subject Classifications (2010): 65M60, 65M08, 35Q35, 76D07

References

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