MULTISCALE HYBRID-MIXED METHOD FOR THE OSEEN EQUATION

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Abstract. This work extends the multiscale hybrid-mixed (MHM) method to the Oseen Equation with highly heterogeneous coefficients. The approach is constructive. Here, we first propose an equivalent dual-hybrid formulation of the original problem using a coarse partition of the heterogeneous domain. Faces may be not aligned with jumps in the data and then, the exact velocity and the pressure are characterized as the solution of a global face problem and the solutions of local independent Oseen problems the continuous level. Owing to this decomposition, the one-level MHM method stems from the standard Galerkin approach for the Lagrange multiplier space. Basis functions are responsible for upscaling the unresolved scales of the medium into the global formulation. They are the exact solution of the local problems with prescribed Neumann boundary conditions on faces driven by the Lagrange multipliers. We apply the MHM method by adopting the unusual stabilized finite element method to solve the local problems approximately. As such, equal order interpolation turns out to be an option for the velocity, the pressure and the Lagrange multipliers. Several academic and highly heterogeneous tests infer that the method achieves super-convergence for the velocity as well optimal convergence for the pressure and also for the stress tensor in their natural norms.

Keywords: Oseen equation, mixed method, hybrid method , multiscale finite element.

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