A DPG FRAMEWORK FOR MONOTONE OPERATORS

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Abstract. We present and analyze a new technique to numerically solve monotone nonlinear problems using the discontinuous Petrov–Galerkin method with optimal test functions. Our strategy is to relax the nonlinear problem to a linear one with additional unknowns and to consider the nonlinear relation as a constraint. We propose to use optimal test functions only for the linear problem and to enforce the nonlinear constraint by penalization. Our scheme can be seen as a minimum residual method with nonlinear penalty term. We prove under appropriate assumptions the well-posedness of the continuous formulation and the quasi-optimal convergence of its discretization. As an application we consider an advection-diffusion problem with nonlinear diffusion of strongly monotone type. Some numerical results in the lowest-order setting are presented to illustrate the predicted convergence.

Keywords: Nonlinear monotone operator, discontinuous Petrov–Galerkin method, ultra-weak formulation

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