FLOTATION WITH SEDIMENTATION I: MODELS WITH SYSTEMS OF CONSERVATION LAWS WITH DISCONTINUOUS FLUX

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Abstract. Flotation is a separation process used in mineral processing to separate out valuable minerals or coal from unwanted decay products, in wastewater treatment to remove unwanted organic solids, dissolved oils and algae, and in chemical engineering contexts. In a flotation column, gas bubbles are injected into a suspension of valuable and unwanted small particles or droplets. Hydrophobic particles attach to the gas bubbles and rise to the top, whereas hydrophilic particles remain in the liquid and may settle therein. A suspension of hydrophilic particles is withdrawn at the bottom, while aggregates of bubbles and hydrophobic particles/droplets are taken out at the top [3]. In mineral processing, wash water is injected at the top of the layer of froth to further remove hydrophilic particles from the aggregates.

The drift-flux theory (stationary conditions) of rising bubbles in a liquid is based on an established constitutive assumption that the flux of rising bubbles is a nonlinear function of the local concentration of bubbles similarly to the Kynch batch-settling flux function in sedimentation. We show that a system of governing conservation PDEs in one dimension, containing these two constitutive functions, can be derived from fundamental principles for the three-phase flow of rising aggregates and settling particles in water with possible attachment of hydrophobic particles to the aggregates. The spatially discontinuous flux functions of the PDEs arise from the three inlets of feed suspension, wash water and gas, and the two outlets.

We have in [1] taken the first step with a scalar model for the two-phase system of bubbles in a fluid. By means of a suitable entropy condition at each flux discontinuity [2] all stationary solutions are categorized. The second step is to analyse the triangular hyperbolic 2×2 system with discontinuous flux modelling the three-phase gas-particle-fluid flow. Solutions can be obtained by first solving the scalar equation for the volume fraction of bubbles for a wished time period and then solving the equation for the volume fraction of particles. All steady state solutions are categorized. A third challenge is to include the attachment process of hydrophobic particles to the bubbles, which leads to a system of three PDEs with discontinuous flux and sources terms.

Keywords: kinematic flow models, flotation, sedimentation, conservation law, discontinuous flux

Mathematics Subject Classifications (2010): 35L65, 35R05, 76T10

References

[1] R. Bürger, S. Diehl, and M.C. Martí, A conservation law with multiply discontinuous flux modelling a flotation column, *Networks and Heterogeneous Media*, **13(2)**: 339–371 2018.

[2] S. Diehl, On scalar conservation laws with point source and discontinuous flux function. *SIAM J. Math. Anal.*, **26(6)**: 1425–1451, 1995.

[3] J. A. Finch and G. S. Dobby, *Column Flotation*, Pergamon Press, London, 1990.

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