ON $\boldsymbol{H}(\mathrm{div})$-CONFORMING METHODS FOR DOUBLE-DIFFUSION EQUATIONS IN POROUS MEDIA

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Abstract. A stationary Navier-Stokes-Brinkman model coupled to a system of advection-diffusion equations serves as a model for so-called double-diffusive viscous flow in porous media in which both heat and a solute within the fluid phase are subject to transport and diffusion. The solvability analysis of these governing equations results as a combination of compactness arguments and fixed-point theory. In addition an $\boldsymbol{H}(\mathrm{div})$-conforming discretisation is formulated by a modification of existing methods for Brinkman flows. The well-posedness of the discrete Galerkin formulation is also discussed, and convergence properties are derived rigorously. Computational tests confirm the predicted rates of error decay and illustrate the applicability of the methods for the simulation of bacterial bioconvection and thermohaline circulation problems.

Keywords: Viscous flow in porous media; doubly-diffusive problems; cross-diffusion; fixed-point theory; mixed finite element methods; a priori error estimation.

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