COMPUTATION OF FRACTIONAL MINIMAL SURFACES

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ABSTRACT. The study of fractional minimal surfaces, which can be interpreted as a non-infinitesimal version of classical minimal surfaces, began with the seminal works [1, 3]. A major difference between local and fractional minimal surface problems is the emergence of *stickiness* phenomena: in general, nonlocal minimal surfaces develop discontinuities across the boundary of the domain [2].

In this talk, we restrict our attention to graph fractional minimal surfaces of order $s \in (0, 1/2)$ on a bounded domain Ω. This graph Plateau problem of order s can be reinterpreted as a Dirichlet problem for a nonlocal, nonlinear, degenerate operator of order $s+1/2$. We propose a finite element method scheme to compute discrete approximations to such minimal surfaces. We prove that this scheme converges in $W^{2t,1}(\Omega)$ for all $t < s$, where $W^{2s,1}(\Omega)$ is the natural energy space. Moreover, we introduce a geometric notion of error, that recovers a weighted L^2 error for the normal vectors in the limit $s \to 1/2$ and for which our method is convergent. In spite of performing approximations with continuous, piecewise linear, Lagrangian finite elements, the stickiness phenomenon becomes apparent in the numerical experiments we present.

Keywords: fractional minimal surfaces, finite element method.

Mathematics Subject Classifications (2010): 65N30, 49F10, 47G10.

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