AN ENTROPY STABLE NODAL DISCONTINUOUS GALERKIN METHOD FOR THE RESISTIVE MHD EQUATIONS

MARVIN BOHM, ANDREW R. WINTERS, GREGOR J. GASSNER, DOMINIK DERIGS, AND FLORIAN HINDENLANG

Abstract. In this talk an extension of discretely entropy stable discontinuous Galerkin (DG) methods to the resistive magnetohydrodynamics (MHD) equations is presented. In the continuous entropy analysis of the ideal MHD equations, which are the advective parts of the resistive MHD equations, the divergence-free constraint on the magnetic field components must be incorporated as a non-conservative term in a form proposed by Powell. Consequently, this non-conservative term needs to be discretized, such that the approximation is consistent with the entropy.

Furthermore, as an extension of the ideal MHD system, the resistive parts are shown to satisfy the entropy inequality, continuously as well as discretely. This enables the construction of an entropy stable DG discretization for the resistive MHD equations.

However, the resulting method suffers from large errors in the divergence-free constraint, since no particular treatment of divergence errors is included in the standard resistive MHD model. Hence, we construct and analyze a DG method that is entropy stable for the resistive MHD equations and has a built-in generalized Lagrange multiplier (GLM) divergence cleaning mechanism. Moreover, the considered scheme is derived for unstructured three-dimensional curvilinear hexahedral meshes.

Finally, we provide numerical verification of the theoretical properties as well as the increased robustness of the derived method compared to standard high-order DG methods. We conclude with some simulation results, in which the novel solver is applied to real space physics MHD flows.

Keywords: resistive magnetohydrodynamics, entropy stability, discontinuous Galerkin spectral element method, hyperbolic divergence cleaning, curvilinear hexahedral mesh, summation-by-parts

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Mathematisches Institut, Universität zu Köln, Weyertal 86-90, 50931 Köln
E-mail address: mbohm@math.uni-koeln.de

Mathematisches Institut, Universität zu Köln, Weyertal 86-90, 50931 Köln
E-mail address: awinters@math.uni-koeln.de

Mathematisches Institut, Universität zu Köln, Weyertal 86-90, 50931 Köln
E-mail address: ggassner@math.uni-koeln.de

I. Physikalisches Institut, Universität zu Köln, Zülpicher Strasse 77, 50937 Köln
E-mail address: derigs@ph1.uni-koeln.de

Max-Planck Institut für Plasmaphysik, Boltzmannstrasse 2, 85748 Garching
E-mail address: florian.hindenlang@ipp.mpg.de