ORTHOGONAL PROJECTION ON POLYNOMIALS AND SOBOLEV-LIKE SPACES WITH REFLECTION-INVARIANT WEIGHTS

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Abstract. We study approximation properties of weighted L^2-orthogonal projectors onto spaces of polynomials of bounded degree in the Euclidean unit ball, where the weight is of the form $x \mapsto (1 - \|x\|^2)^\alpha \prod_{i=1}^d |x_i|^{\gamma_i}$, $\alpha > -1$, $\gamma \in (-1, \infty)^d$. Such weights are invariant under the abelian group $\mathbb{Z}_2^d$ and are, in general, singular in the interior of their domain. Said properties are measured in Sobolev-type norms in which the same weighted L^2 norm is used to control all the involved Dunkl differential-difference operators. Those operators reduce to partial derivatives for functions with $\mathbb{Z}_2^d$-symmetry.

The method of proof does not rely on any particular basis of orthogonal polynomials, which allows for a streamlined and dimension-independent exposition. The obtained results and their methods of proof are prototypes of approximation theoretical advances relevant to the Numerical Analysis of problems with interior singularities.

Keywords: Orthogonal projection; Weighted Sobolev space; Unit ball; Orthogonal polynomials; Reflection-invariant weights.

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