WELL-POSEDNESS RESULT FOR AN HYPERBOLIC SCALAR CONSERVATION LAW WITH A STOCHASTIC FORCE USING A FINITE VOLUME APPROXIMATION

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Abstract. We are interested in the Cauchy problem for a nonlinear hyperbolic scalar conservation law in d space dimensions forced by a multiplicative stochastic noise (in the sense of Itô) and with a general time and space dependent flux-function of type:

$$\left\{\begin{array}{rcll} du+\operatorname{div}_x\left[\vec{\mathbf{f}}(x,t,u)\right]dt &=& g(u)dW & \text{in } \Omega\times\mathbb{R}^d\times(0,T), \\ u(\omega,x,0) &=& u_0(x), & \omega\in\Omega,\ x\in\mathbb{R}^d. \end{array}\right. \tag{1}$$

Here d is a positive integer, $T>0$, div_x is the divergence operator with respect to the space variable (which belongs to $\mathbb{R}^d$) and $W=\{W_t,\mathcal{F}_t; 0\leqslant t\leqslant T\}$ is a standard adapted one-dimensional continuous Brownian motion defined on the classical Wiener space $(\Omega,\mathcal{F},\mathbb{P})$. The well-posedness theory for (1) is a known result since the work of [DeV10] by the way of a kinetic approach and the numerical approximation of its solution has been the subject of many work recently ([KR12, BCG16, M18]...). Our aim here is to address simultaneously theoretical and numerical issues. More precisely I will present an existence and uniqueness result of the stochastic entropy solution of (1) together with the convergence of a finite volume approximation. Comparing with the existing results on the subject, the true novelty of the present study is the use of the numerical approximation to get both existence and uniqueness of the stochastic entropy solution.

My talk will be separated in three parts. In a first one I will introduce the notion of stochastic entropy solution for (1) as well as a generalized notion of solution (namely measure-valued entropy solution). In a second one, I will present the monotone finite volume scheme used to approximate our problem and show its convergence towards a measure-valued entropy solution. The last part will be devoted to show the uniqueness of this generalized solution as well as the existence and uniqueness of the stochastic entropy solution. The idea is to adapt the Kruzkhov doubling variable technique to the stochastic case by comparing any generalized solution to the finite volume approximation.

Keywords: Stochastic PDE, multiplicative noise, finite volume method, monotone scheme, entropy solution, doubling variable method, Young measures, measure-valued entropy solution.

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