A HYBRIDIZABLE DISCONTINUOUS GALERKIN METHOD FOR THE BIHARMONIC PROBLEM

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Abstract. A Hybridizable Discontinuous Galerkin (HDG) method for solving the biharmonic problem $\Delta^2 u = f$ is proposed and analyzed in this work. More precisely, we employ an HDG method based on a system of first-order equations, which we propose to approximate u, ∇u, $\mathcal{H}(u)$ and $\nabla \cdot \mathcal{H}(u)$ simultaneously, where $\mathcal{H}$ correspond to Hessian matrix. This method allows us to eliminate all the interior variables locally to obtain a global system for $\hat{u}_h$ and $\hat{\boldsymbol{q}}_h$ that approximate u and ∇u, respectively, on the interfaces of the triangulation. As a consequence the only globally coupled degrees of freedom are those of the approximations to u and ∇u on the faces of the elements. We also carry out an priori error analysis using the orthogonal L^2-projector and concluded that the orders of convergence for the errors in the approximation of $\mathcal{H}(u)$, $\nabla \cdot \mathcal{H}(u)$, ∇u and u are $h^{k+1/2}$, $h^{k-1/2}$, h^k, y h^{k+1} respectively, where $k \geq 1$ is the polynomial degree of the local spaces and h is the meshsize. Our numerical results suggest that our estimate for u is sharp, whereas the approximations of $H(u)$, ∇u and $\nabla \cdot \mathcal{H}(u)$ converge with an extra half a power of h .

Keywords: Hybridizable Discontinuous Galerkin, Biharmonic, High order

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