LOW-ORDER DIVERGENCE-FREE FINITE ELEMENT METHODS IN FLUID MECHANICS

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Abstract. It is a well-known fact that the finite element approximation of equations in fluid mechanics (e.g., the Navier-Stokes equations) is simpler, and more accurate, if the finite element method delivers an exactly divergence-free velocity. In fact, in such a case the convective term remains antisymmetric in the discrete setting, without the need to rewrite it, and the stability analysis can be greatly simplified. Now, when the finite element spaces used are the conforming $\mathbb{P}_1$ for velocity and $\mathbb{P}_0$ for pressure, then, in addition to add appropriate terms to stabilise the pressure, some extra work needs to be done in order to compensate for the lack of incompressibility of the discrete velocity.

In this talk I will present two recent applications of a technique developed previously in [2] to overcome the limitations described in the previous paragraph. The first example presented is the application of this idea to the steady-state Boussinesq equation [1]. Then, the transient Navier-Stokes equations will be analysed, where some error estimates independent of the Reynolds number will be shown.

Keywords: Multiscale finite element method; hybrid formulation; polygonal mesh; optimal convergence.

Mathematics Subject Classifications (2010): 65N15

References

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