CONVERGENCE OF A LEVEL-SET ALGORITHM FOR SCALAR CONSERVATION LAWS

ANÍBAL CORONEL AND MAURICIO SEPÚLVEDA

Abstract. In this paper we study the convergence of the level-set algorithm introduced by Aslam for tracking the discontinuities in scalar conservation laws in the case of linear or strictly convex flux function (2001, *J. Comput. Phy.* 167, 413-438). The numerical method is deduced by the level-set representation of the entropy solution: the zero of a level-set function is used as an indicator of the discontinuity curves and two auxiliary states, which are assumed continuous through the discontinuities, are introduced. Following the ideas of (2015 *Numer. Meth. for PDE* 31, 1310–1343), we rewrite the numerical level-set algorithm as a procedure consisting of three big steps: (a) initialization, (b) evolution and (c) reconstruction. In (a) we choose an entropy admissible level-set representation of the initial condition. In (b), for each iteration step, we solve an uncoupled system of three equations and select the entropy admissible level-set representation of the solution profile at the end of the time iteration. In (c) we reconstruct the entropy solution by using the level-set representation. Assuming that in the step (b) we can use a second order scheme to approximate each equation of that we prove the convergence of the numerical solution of the level set algorithm to the entropy solution in L^1, using the ideas of Popov and collaborators (2005 *SIAM J. Numer. Anal.* 42, 1978–1997; 2006 *Numer. Math.* 104, 539–559; and 2006 *Math. Comp.* 75, 1735–1753). In addition, some numerical examples focused on the elementary wave interaction are presented.

Keywords: conservation laws; convergence of finite volume methods; level sets

Mathematics Subject Classifications (2010): 65M08, 65M12

References

[1] T. D. Aslam, A level set algorithm for tracking discontinuities in hyperbolic conservation laws I: scalar equations, *J. Comput. Phys.* 167(2):413–438, 2001.

[2] A. Coronel, P. Cumsille and M. Sepúlveda. Convergence of a level-set algorithm in scalar conservation laws. *Numerical Methods Partial Differential Equations*, 31(4):1310–1343, 2015.

[3] S. Konyagin, B. Popov, and O. Trifonov, Ognian . On convergence of minmod-type schemes. SIAM *J. Numer. Anal.* 42(5):1978–1997, 2005.

[4] B. Popov and O. Trifonov. One-sided stability and convergence of the Nessyahu-Tadmor scheme. *Numer. Math.* 104(4):539–559, 2006.

[5] B. Popov and O. Trifonov. Order of convergence of second order schemes based on the minmod limiter. *Math. Comp.* 75(256):1735–1753, 2006.

GMA, Departamento de Ciencias Básicas, Facultad de Ciencias, Universidad del Bío-Bío, Campus Fernando May, Chillán, Chile,
E-mail address: acoronel@ubiobio.cl

CI^{2}MA and DIM, Universidad de Concepción, Concepción, Chile.
E-mail address: mauricio@ing-mat.udec.cl