CONVERGENCE OF A LEVEL-SET ALGORITHM FOR SCALAR
CONSERVATION LAWS

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ABSTRACT. In this paper we study the convergence of the level-set algorithm introduced by Aslam for tracking the discontinuities in scalar conservation laws in the case of linear or strictly convex flux function (2001, J. Comput. Phy. 167, 413-438). The numerical method is deduced by the level-set representation of the entropy solution: the zero of a level-set function is used as an indicator of the discontinuity curves and two auxiliary states, which are assumed continuous through the discontinuities, are introduced. Following the ideas of (2015 Numer. Meth. for PDE 31, 1310–1343), we rewrite the numerical level-set algorithm as a procedure consisting of three big steps: (a) initialization, (b) evolution and (c) reconstruction. In (a) we choose an entropy admissible level-set representation of the initial condition. In (b), for each iteration step, we solve an uncoupled system of three equations and select the entropy admissible level-set representation of the solution profile at the end of the time iteration. In (c) we reconstruct the entropy solution by using the level-set representation. Assuming that in the step (b) we can use a second order scheme to approximate each equation of that we prove the convergence of the numerical solution of the level set algorithm to the entropy solution in $L^1$, using the ideas of Popov and collaborators (2005 SIAM J. Numer. Anal. 42, 1978–1997; 2006 Numer. Math. 104, 539–559; and 2006 Math. Comp. 75, 1735–1753). In addition, some numerical examples focused on the elementary wave interaction are presented.

Keywords: conservation laws; convergence of finite volume methods; level sets

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REFERENCES


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