THE "FAST HYBRID METHOD": TECHNIQUES FOR EFFICIENT AND PARALLELIZABLE, DISPERSIONLESS, HIGH-ORDER SOLUTION OF WAVE SCATTERING PROBLEMS

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Abstract. We propose a frequency/time hybrid integral-equation method for the time dependent wave equation in two and three-dimensional spatial domains. Relying on Fourier Transformation in time, the method utilizes a fixed (time-independent) number of frequency-domain integral-equation solutions to evaluate, with superalgebraically-small errors, time domain solutions for arbitrarily long times. The approach relies on two main elements, namely, 1) A smooth time-windowing methodology that enables accurate band-limited representations for arbitrary long time signals, and 2) A novel Fourier transform approach which, in a time-parallel manner and without causing spurious periodicity effects, delivers numerically dispersionless spectrally accurate solutions. A similar hybrid technique can be obtained on the basis of Laplace transforms instead of Fourier transforms. We will demonstrate results in two- and three-dimensional scattering, as well as results with dispersive media, a challenging problem for other transient solution approaches. The algorithm can tackle complex physical structures, it enables parallelization in time in a straightforward manner, and it allows for time leaping—that is, solution sampling at any given time T at $\mathcal{O}(1)$-bounded sampling cost, for arbitrarily large values of T, and without requirement of evaluation of the solution at intermediate times. The proposed frequency/time hybridization strategy, which generalizes to any linear partial differential equation in the time domain for which frequency-domain solutions can be obtained (including e.g. the time-domain Maxwell equations), and which is applicable in a wide range of scientific and engineering contexts, provides significant advantages over other available alternatives such as volumetric discretization and convolution-quadrature approaches.

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References

[1] C. Lubich, *On the multistep time discretization of linear initial-boundary value problems and their boundary integral equations*, Numerische Mathematik, 67 (1994), pp. 365–389.

[2] E. Mecocci, L. Misici, M. C. Recchioni, and F. Zirilli, *A new formalism for time-dependent wave scattering from a bounded obstacle*, The Journal of the Acoustical Society of America, 107 (2000), pp. 1825–1840.

[3] L. Banjai and S. Sauter, *Rapid solution of the wave equation in unbounded domains*, SIAM Journal on Numerical Analysis, 47 (2009), pp. 227–249.

[4] V. Nascov and P. C. Logofătu, *Fast computation algorithm for the rayleigh-sommerfeld diffraction formula using a type of scaled convolution*, Appl. Opt., 48 (2009), pp. 4310–4319.

[5] T. Betcke, N. Salles, and W. Śmigaj, *Overresolving in the laplace domain for convolution quadrature methods*, SIAM Journal on Scientific Computing, 39 (2017), pp. A188–A213

[6] O. P. Bruno and E. Garza, *A Chebyshev-based rectangular-polar integral solver for scattering by general geometries described by non-overlapping patches*, 2018.

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