

# ON A MIXED METHOD FOR THE OBERBECK-BOUSSINESQ MODEL OF HEAT AND MASS TRANSFER

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ABSTRACT. Natural convection is of paramount interest since it appears in many environmental processes and diverse industrial practical applications. In this talk, I will present a work in progress that extends a previous mixed finite element method for heat driven flows to the more complex and realistic situation in which this phenomena results from bouyancy forces generated not only by temperature differences but also species concentration effects. The model in consideration is written in the Oberbeck-Boussinesq approximation framework; a system given by the incompressible Navier-Stokes equations, and the advection-diffusion equations for describing the underlying hydrodynamic, and the substance concentration and the temperature, respectively. The main features of the approach and the continuous and discrete solvability results obtained so far for the corresponding variational formulation will be presented along with some preliminary numerical experiments suggesting optimal a priori error estimates.

**Keywords:** mixed finite element method, Oberbeck-Boussinesq equations, a priori error estimates.

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