NEW MIXED FINITE ELEMENT METHODS FOR NATURAL CONVECTION WITH PHASE-CHANGE IN POROUS MEDIA

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Abstract. This article is concerned with the mathematical and numerical analysis of a steady phase change problem for non-isothermal incompressible viscous flow. The system is formulated in terms of pseudostress, strain rate and velocity for the Navier-Stokes-Brinkman equation, whereas temperature, normal heat flux on the boundary, and an auxiliary unknown are introduced for the energy conservation equation. In addition, and as one of the novelties of our approach, the symmetry of the pseudostress is imposed in an ultra-weak sense, thanks to which the usual introduction of the vorticity as an additional unknown is no longer needed. Then, for the mathematical analysis two variational formulations are proposed, namely mixed-primal and fully-mixed approaches, and the solvability of the resulting coupled formulations is established by combining fixed-point arguments, Sobolev embedding theorems and certain regularity assumptions. We then construct corresponding Galerkin discretizations based on adequate finite element spaces, and derive optimal a priori error estimates. Finally, numerical experiments in 2D and 3D illustrate the interest of this scheme and validate the theory.

Keywords: Natural convection, viscous flow in porous media, change of phase, mixed–primal formulation, fully–mixed formulation, fixed-point theory, finite element methods.

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