AN AFEM FOR THE POINTWISE TRACKING OPTIMAL CONTROL PROBLEM OF THE STOKES EQUATIONS

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Abstract. We propose and analyze a reliable and efficient a posteriori error estimator for the pointwise tracking optimal control problem of the Stokes equations. This linear–quadratic optimal control problem entails the minimization of a cost functional that involves point evaluations of the velocity field that solves the state equations. This leads to an adjoint problem with a linear combination of Dirac measures as a forcing term and whose solution exhibits reduced regularity properties. We also consider constraints on the control variable. The proposed a posteriori error estimator can be decomposed as the sum of four contributions: three contributions related to the discretization of the state and adjoint equations, and another contribution that accounts for the discretization of the control variable. On the basis of the devised a posteriori error estimator, we design a simple adaptive strategy that illustrates our theory and exhibits a competitive performance.

Keywords: linear-quadratic optimal control problems, Stokes equations, a posteriori error estimates, Dirac measures, Muckenhoupt weights, maximum–norm.

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