ADJOINT METHODS FOR TIME-DEPENDENT HYPERBOLIC PDES AND APPLICATIONS TO TSUNAMI MODELING

BRISA N. DAVIS AND RANDALL J. LEVEQUE

ABSTRACT. For time-dependent hyperbolic partial differential equations that model wave propagation, the adjoint problem is a closely related hyperbolic PDE that must be solved backwards in time. There are several uses for an adjoint solution, including as a tool to determine where adaptive mesh refinement (AMR) should be applied, and for sensitivity studies or the solution of inverse problems.

A general approach for using the adjoint in the context of AMR has recently been implemented in the Clawpack software [2] for the case of linear PDEs such as acoustics or linear elasticity (e.g. for seismic wave propagation), and also in the GeoClaw subset of this software that is widely used for tsunami modeling [1, 3, 4, 5]. Clawpack provides high-resolution (shock-capturing) finite volume methods for solving hyperbolic problems based on a Riemann solver for the specific PDE. Adaptive refinement introduces a hierarchy of finer grid patches in regions where the solution must be captured on a finer grid. These patches can be adjusted every few time steps to follow propagating waves.

For many problems the primary interest is in tracking waves that reach one target location, perhaps after multiple reflections. The solution to an adjoint equation solved backward in time from the target location can be used to identify the regions that require refinement. In particular, the adjoint solution can be used to help track tsunami waves in the ocean that will reach a particular community of interest. This can lead to orders of magnitude speed up relative to refining the entire ocean. The adjoint can also help identify what combination of waves arrive simultaneously and give particularly large run-up, and when inverting to determine the source that created an observed tide gauge signal. Several examples will be shown of applications to tsunami modeling.

Keywords: hyperbolic PDEs, finite volume methods, adjoint equation, adaptive refinement, tsunami modeling.

Mathematics Subject Classifications (2010): 65M08

References

[1] J. C. Borrero, R. J. LeVeque, S. D. Greer, S. O'Neill, and B. N. Davis, *Modelling tsunami currents at the Port of Tauranga, New Zealand*, in Australasian Coasts & Ports Conference 2015, J. C. Borrero et al., eds., Engineers Australia and IPENZ, pp. 90–95.
http://faculty.washington.edu/rjl/pubs/Tauranga2015/.

[2] Clawpack Developers, *Clawpack Software*. http://www.clawpack.org.

[3] B. N. Davis, *Adjoint-Guided Adaptive Mesh Refinement for Hyperbolic Systems of Equations*, PhD thesis, University of Washington, 2018,
http://faculty.washington.edu/rjl/students/davis/BNDavis_uwthesis.pdf.

[4] B. N. Davis and R. J. LeVeque, *Adjoint Methods for Guiding Adaptive Mesh Refinement in Tsunami Modeling*, Pure and Applied Geophysics, 173 (2016), pp. 4055–4074,
https://doi.org/10.1007/s00024-016-1412-y.

[5] B. N. Davis and R. J. LeVeque, *Analysis and performance evaluation of adjoint-guided adaptive mesh refinement for linear hyperbolic pdes using clawpack*. Submitted, 2018, http://arxiv.org/abs/1810.00927.

Department of Mathematics, LeTourneau University
E-mail address: brisadavis@letu.edu

Department of Applied Mathematics, University of Washington
E-mail address: rjl@uw.edu