OPTIMAL WAVELET SOLVERS FOR CONTROL PROBLEMS CONSTRAINED BY EVOLUTION PDES

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Abstract. Numerical solvers for PDEs have matured over the past decades in efficiency, largely due to the development of sophisticated algorithms based on closely intertwining theory with numerical analysis. Consequently, systems of PDEs as they arise from optimization problems with PDE constraints also have become more and more tractable. For constraints in form of a time-dependent PDE, the first-order necessary conditions for optimality require to repeatedly solve a system of PDEs for the unknowns (state, costate and control) which is coupled globally in space and time. For these, conventional time-marching methods quickly reach computational limitations due to the massive requirement of storage. To overcome this problem, adaptive methods which judiciously distribute degrees of freedom with respect to both space and time during the computations to resolve singularities in the data and/or domain appear to be most promising.

Specifying the situation to control problems constrained by a linear evolution PDE of parabolic type, I start out by formulating the constraint in a weak space-time form. This is the basis for the design of an adaptive wavelet method for which convergence and optimal convergence rates (when compared to wavelet-best N-term approximations, a concept from nonlinear approximation theory) can be shown. In addition, wavelets as discretization ingredients allow for fast solvers of optimal linear complexity. Furthermore, the even more difficult and recently increasingly more important class of parameter-dependent control problems depending on even possibly countable many parameters can be attacked in this framework.

I would like to conclude with some remarks concerning corresponding finite element discretizations.

The results were obtained in collaboration with Max Gunzburger (Florida State University) and with Christoph Schwab (ETH Zürich).

Keywords: Linear-quadratic optimal control problems, parabolic PDE, adaptivity, wavelets, convergence, optimal complexity.

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