

# WAVELETS MEET BOUNDARY INTEGRAL EQUATIONS

HELMUT HARBRECHT

ABSTRACT. Many mathematical models, concerning for example field calculations, flow simulation, elasticity or visualization, are based on operator equations with nonlocal operators, especially boundary integral operators. Solving such equations by the Galerkin method leads to densely populated system matrices which are often ill conditioned. Thus, the memory consumption and the work to compute the solution scales at least quadratically. This makes the boundary element method unattractive for the practical usage.

In recent years, algorithms like the Fast Multipole Method and the Adaptive Cross Approximation have been developed to reduce the complexity considerably. Another efficient method is the wavelet Galerkin method: one employs biorthogonal wavelet bases with vanishing moments for the discretization of the given boundary integral equation. The resulting system matrix is quasi-sparse and can be compressed without loss of accuracy such that linear over-all complexity is realized.

This talk is concerned with the principles as well as new developments of the wavelet Galerkin method for boundary integral equations, particularly assembling the compressed system matrix, preconditioning, and adaptivity. Numerical experiments are presented which complement the theory: The matrix compression does not compromise the accuracy of the Galerkin method. However, we save a factor of storage 100–1000 and accelerate the computing time up to a factor 100.

**Keywords:** boundary element method, wavelet matrix compression, adaptivity

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DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE, UNIVERSITY OF BASEL, SWITZERLAND  
*E-mail address:* `helmut.harbrecht@unibas.ch`