ILL-POSED PROBLEMS AND STABILIZED FINITE ELEMENT METHODS

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ABSTRACT. In this talk we will consider some ill-posed elliptic equations and their discretization using finite element methods. The standard approach to ill-posed problems is to regularize the continuous problem so that existence and uniqueness is guaranteed. The regularized problem can then be solved using standard finite element methods. When using this strategy, in order to optimize accuracy, the regularization parameter must be chosen as a function both of the stability properties of the ill-posed problem, the mesh parameter and perturbations in data. Here we will propose a different approach [1], where the ill-posed pde is discretized in an optimization framework, prior to regularization. To ensure discrete well-posedness we add stabilizing terms to the formulation, drawing on experience from stabilized FEM and discontinuous Galerkin methods. The error in the resulting finite element reconstructions is then analyzed using Carleman estimates on the continuous problem. This results in approximations that are optimal with respect to the approximation order of the finite element space and the stability of the computed quantity. The mesh parameter here plays the role of the regularization parameter. Mesh resolution can be chosen independently of the stability properties of the physical problem, but must match perturbations in data, in a way made explicit in the estimates. Some examples of problems analyzed in this framework will be presented, selected from recent work on the Helmholtz equation [4], the convection–diffusion equation [5], Stokes' equations [2] and Darcy's equation [3].

Keywords: ill-posed problems, data assimilation, stabilized finite element methods

Mathematics Subject Classifications (2010): 65N30, 35R25, 65N20

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