AN INTRODUCTION TO VIRTUAL ELEMENTS IN 3D

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Abstract. The Virtual Element Method (VEM), is a very recent technology introduced in [1] for the discretization of partial differential equations, that has shared a good success in recent years. The VEM can be interpreted as a generalization of the Finite Element Method that allows to use general polygonal and polyhedral meshes, still keeping the same coding complexity and allowing for arbitrary degree of accuracy. The Virtual Element Method makes use of local functions that are not necessarily polynomials and are defined in an implicit way. Nevertheless, by a wise choice of the degrees of freedom and introducing a novel construction of the associated stiffness matrixes, the VEM avoids the explicit integration of such shape functions. In addition to the possibility to handle general polytopal meshes, the flexibility of the above construction yields other interesting properties with respect to more standard Galerkin methods (for instance, the VEM easily allows to build discrete spaces of arbitrary C^k regularity, or to satisfy exactly the divergence-free constraint for incompressible fluids).

The present talk is an introduction to the VEM, aiming at showing the main ideas of the method. We consider for simplicity a simple elliptic model problem (that is pure diffusion with variable coefficients [2]) but set ourselves in the more involved 3D setting [3]. In the first part we introduce the adopted Virtual Element space and the associated degrees of freedom, first by addressing the faces of the polyhedrons (i.e. polygons) and then building the space in the full volumes. We then describe the construction of the discrete bilinear form and the ensuing discretization of the problem. Furthermore, we show a set of theoretical and numerical results. In the very final part, we will give a glance at more involved problems, such as magnetostatics (mixed problem with more complex spaces interacting) and large deformation elasticity (nonlinear problem).

Keywords: Finite Element Method, Virtual Element Method, Polyhedral meshes.

Mathematics Subject Classifications (2010): 65N30.

References

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