GUARANTEED AND ROBUST A POSTERIORI BOUNDS FOR LAPLACE EIGENVALUES AND EIGENVECTORS

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Abstract. We derive a posteriori error estimates for the Laplace eigenvalue problem with a homogeneous Dirichlet boundary condition. First, upper and lower bounds for the first eigenvalue are given. These bounds are guaranteed, fully computable, and converge with the optimal speed to the exact eigenvalue. They are valid under explicit, a posteriori conditions on the computational mesh and on the approximate solution. Guaranteed, fully computable, and polynomial-degree robust bounds for the energy error in the approximation of the first eigenvector are derived as well, under the same conditions. Remarkably, all the constants in our theory can be fully estimated, and no convexity/regularity assumption on the computational domain/exact eigenvector(s) is needed. This general result can still be improved when an elliptic regularity assumption is satisfied (with known constants), typically for convex two-dimensional domains. The application of our framework to conforming, nonconforming, discontinuous Galerkin, and mixed finite element approximations of arbitrary polynomial degree is provided, along with a numerical illustration on a set of test problems.

References

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