SEMI-IMPLICIT IMEX SCHEMES FOR EVOLUTIONARY PDE'S

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Abstract. In the talk we consider a new formulation of implicit-explicit (IMEX) Runge-Kutta methods for the numerical discretization of time dependent partial differential equations. The approach is based on identifying the (possibly linear) dependence on the unknown of the system which generates the stiffness. Only the stiff dependence is treated implicitly, then making the whole method much simpler than fully implicit ones. This approach generalizes classical IMEX methods based on additive and partitioned R-K, and allows the construction of new semi-implicit schemes. We present several semi-implicit R-K methods up to order three, and illustrate their effectiveness in several examples [1]. Next we present two non trivial applications of this approach. The first concerns the solution of a class systems of nonlinear convection-diffusion equations (with strongly degenerate diffusion) arising in multispecies kinematic flow models. Such systems describe models of polydisperse sedimentation and multiclass traffic flow [2]. A detailed comparison is performed between classic IMEX-RK schemes, in which the stiff term is treated fully-implicitly, and semi-implicit IMEX-RK, in which the stiff term is treated only linearly implicit. Although IMEX-RK with fully implicit treatment give better resolution near discontinuities, semi-implicit schemes appear to be more cost effective in most cases. A second challenging problem concerns the development of all-Mach number solvers for the Euler equations of gas dynamics. Isentropic and full Euler equations for a polytropic gas are considered, both in one and two space dimensions [3]. Space is discretized on a staggered grid, this avoiding relying on a numerical flux function and on (exact or approximate) Riemann solvers. Second order schemes, both in space and time, are derived, which are able to solve the equation on a wide range of Mach number. The results are compared with those available in the literature, and show the robustness and simplicity of the new approach. Some consideration on the construction of higher order schemes will be given.

Keywords: Implicit-Explicit schemes, low Mach number, nonlinear degenerate diffusion

Mathematics Subject Classifications (2010): 65M20, 65L04, 65L06, 35L65, 76T20

References

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