

# ADVANCES IN DPG METHODS

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**ABSTRACT.** A finite element mesh of a domain can not only be used to break the domain into elements, but also to break a Sobolev space. Any standard Sobolev space has a “broken” analogue obtained by removing all interelement continuity constraints from its functions. This process, often called hybridization, has been well studied within a discrete setting, e.g., the process of discretizing first and then hybridizing is the basis for hybrid mixed methods. If one reverses the order to first hybridize and then discretize, new DPG (Discontinuous Petrov Galerkin) methods can be obtained. These methods were introduced in [2, 3]. DPG methods are conforming within broken Sobolev spaces. The talk will review the various interpretations of DPG methods, summarize the currently known basic properties of these methods, and provide pointers to ongoing research. The talk will also present recent techniques [1] that allow one to conclude stability of certain formulations using the broken spaces from the stability of analogues that use unbroken spaces.

**Keywords:** hybridization, Maxwell equations, stability, wellposedness, inf-sup condition

**Mathematics Subject Classifications (2010):** 65N30

## REFERENCES

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