

# FINITE ELEMENT QUASI-INTERPOLATION AND BEST APPROXIMATION

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ABSTRACT. We devise and analyze quasi-interpolation operators for scalar- and vector-valued finite element spaces constructed on affine, shape-regular meshes with some continuity across mesh interfaces. These operators are  $L^1$ -stable, leave the corresponding finite element space point-wise invariant, whether homogeneous boundary conditions are imposed or not, and, assuming regularity in the fractional Sobolev spaces  $W^{s,p}$  where  $p \in [1, \infty]$  and  $s$  can be arbitrarily close to zero, they give optimal local approximation estimates in any  $L^p$ -norm. The theory is illustrated on  $H^1$ -,  $H(\text{curl})$ - and  $H(\text{div})$ -conforming spaces. More details can be found in [1, 2].

**Keywords:** Quasi-interpolation, Finite Elements, Best Approximation

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## REFERENCES

- [1] A. Ern and J.-L. Guermond. Finite element quasi-interpolation and best approximation. Technical Report <http://arxiv.org/abs/1505.06931>, arXiv, 2015.
- [2] Mollification in strongly Lipschitz domains with application to continuous and discrete de Rham complex. Technical Report <http://arxiv.org/abs/1509.01325>, arXiv, 2015. To appear in *Comp. Methods Appl. Math.*

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