CONVERGENCE ANALYSIS OF A SHOCK-CAPTURING DISCONTINUOUS GALERKIN SCHEME FOR HYPERBOLIC CONSERVATION LAWS

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Abstract. The design and analysis of numerical schemes for nonlinear systems of conservation laws is made difficult by the fact that there is no global well-posedness theory for most interesting problems. Regardless of the particular type of numerical scheme under investigation, the scope of the analysis often restricted to stability estimates. Stronger proofs of convergence are difficult to come by and often hinge on an appropriate notion of convergence and the concept of solution.

Entropy-measure-valued (EMV) solutions [1] are a generalization of traditional entropy weak solutions to conservation laws. It is a classical result for generic scalar conservation laws that EMV solutions reduce to traditional entropy solutions if the initial data is a Dirac measure. For multidimensional systems such general results are not available. Numerical evidence casts some doubt on that entropy solutions constitute the appropriate solution paradigm for this case, and it has been conjectured that MV solutions ought to be considered the appropriate notion of solution. Building on previous results in [2], we prove (c.f. [4]) that bounded solutions of a certain class of space-time Discontinuous Galerkin (DG) schemes converge to an EMV solution.

The novelty in our work is that no streamline-diffusion (SD) terms are used for stabilization. While streamline diffusion stabilization is often included in the analysis of DG schemes (see [2] and references cited therein), it is not commonly found in practical implementations. We show that a properly chosen nonlinear shock-capturing operator suffices to provide the necessary stability and entropy consistency estimates. In the case of scalar equations this result can be strengthened as in [5]; one can show the $L_\infty(L_\infty)$-boundedness of the solutions, without using a finer auxiliary triangulation like [3], and hence the convergence to the unique entropy weak solution is obtained. Our results are valid for arbitrary degree of polynomial approximation.

Keywords: Conservation Law, Discontinuous Galerkin, Convergence Analysis, Shock Capturing

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