# EFFICIENT BOUNDARY ELEMENT METHODS FOR FOCUSED ULTRASOUND TREATMENT PLANNING

ELWIN VAN 'T WOUT, TIMO BETCKE, PIERRE GÉLAT, AND SIMON ARRIDGE

Abstract. Medical therapies with high-intensity focused ultrasound (HIFU) have promising applications in the non-invasive treatment of a wide range of cancers. Yet, their clinical use for diseased tissue in the transcostal region is limited due to the challenges in focussing the ultrasound arrays behind the ribs. Sufficient energy has to be transmitted inside the ribcage to achieve ablation of the diseased tissue while preventing the occurrence of side lobes at the focal region and skin burns at the ribs. The patient-specific planning for transcostal HIFU treatment is likely to rely on numerical modelling to optimise the multi-element array of ultrasound transducers. This necessitates the computation of acoustic scattering at MHz frequencies.

Modern boundary element methods (BEM) provide fast and accurate computation of large-scale scattering characteristics. However, the applicability of BEM to the simulation of high-frequency scattering is limited by the large demand for computational resources, the weak convergence, and the presence of spurious resonance modes. We will present the use of a Burton-Miller formulation in combination with $\mathcal{H}$-matrix compression and high-frequency preconditioning based on on-surface radiation conditions. When heterogeneous regions are present in the model, this fast BEM technique can be coupled with a finite element method (FEM). The open-source library `BEM++` has been used as implementation platform, with FEM-BEM coupled simulations achieved by a combination with the `FEniCS` package. The feasibility of BEM simulations for medical applications is demonstrated with scattering analysis of a human ribcage by focused ultrasound of 1 MHz.

**Keywords**: boundary element methods, finite element methods, preconditioning, acoustic scattering, medical physics, open-source software

**Mathematics Subject Classifications (2010)**: 45P05, 65F08, 78A45, 92C50

## References

[1] E. van 't Wout, P. Gélat, T. Betcke, and S. Arridge. A fast boundary element method for the scattering analysis of high-intensity focused ultrasound. *Journal of the Acoustical Society of America*, vol. 138, 2015.

Pontificia Universidad Católica de Chile, Ingeniería Matemática, Santiago, Chile
*E-mail address*: elwin.puc@gmail.com

University College London, Department of Mathematics, London, United Kingdom
*E-mail address*: t.betcke@ucl.ac.uk

University College London, Department of Mechanical Engineering, London, United Kingdom
*E-mail address*: p.gelat@ucl.ac.uk

University College London, Centre for Medical Image Computing, London, United Kingdom
*E-mail address*: s.arridge@ucl.ac.uk