

# FLUX SPLITTING SCHEMES FOR THE MODEL EQUATIONS OF COMPRESSIBLE MULTIPHASE FLOW

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ABSTRACT. We study the flux vector splitting schemes applied to various model equations of multiphase flow.

First, we extend the flux vector splitting approach proposed recently in [1, 2] for the Euler equations to the case of the Baer-Nunziato equations of compressible two-phase flow with general equation of state. Flux vector splitting schemes provide upwinding for fast waves at a lower computational effort and algorithm complexity than the Godunov approach with a good Riemann solver.

The Baer-Nunziato equations are hyperbolic, except for some well identified situations, and the complete mathematical structure of the 1D system is available [3]. However, the equations cannot be written in conservation-law form due to the presence of nonconservative products and hence the classical Rankine-Hugoniot conditions are not available which imposes additional mathematical and numerical challenges to the numerical solution of such system as well as to the formulation of the flux splitting scheme. The splitting of the conservative flux of the Baer-Nunziato model that we propose is based on the separation of the advection terms from the pressure terms, where the corresponding pressure system includes nonconservative products. We show that the pressure system is hyperbolic and derive its eigenstructure. Next, we construct an approximate Riemann solver to obtain the Godunov states for the pressure system which are then used in the approximation of the convective part of the flux and the nonconservative terms in the finite-volume framework.

Second, we attempt to apply the flux splitting scheme for the 5-equation reduced model of compressible multiphase flow [4].

**Keywords:** hyperbolic systems, flux vector splitting, Baer-Nunziato equations, reduced model, multiphase flow

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