HOW TO AVOID MASS MATRIX INVERSION IN CONTINUOUS FINITE ELEMENTS: APPLICATION TO LAGRANGIAN HYDRODYNAMICS

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Abstract. We revise and improve the high-order finite element scheme for Lagrangian hydrodynamics proposed in [1]. We adopt the staggered grid discretization of the unknowns: the kinematic variables are approximated in a continuous finite-element space with node-based shape functions while the thermodynamic variables are represented using a discontinuous Galerkin formulation with cell-based shape functions. The main drawbacks of the original mixed scheme are (1) the need to invert a global sparse mass matrix induced by the FE approximation and (2) the need of artificial viscosity term for stabilization. To overcome these drawbacks, we develop a high-order residual distribution scheme which allows an efficient mass lumping, i. e. diagonalization of the mass matrix without a loss of high-order accuracy and doesn't require any artificial viscosity for FEM stabilization.

To achieve this, we interpret the original finite-element scheme as a residual distribution scheme and propose a simple technique to avoid the inversion of the mass matrix when using continuous finite elements. This technique employs two main steps. The first one allows to simplify the nonlinear residual based stabilization and express it only in terms of known values of the solution. This leads to the removal of the nonlinear and non-symmetric part of the mass matrix. The second step consists in finding a constraint on the type of finite element approximation used, and in particular on the nature of the associated lumped Galerkin mass matrix. These two steps can be integrated both in multistage and multistep time marching procedures. A generalisation can be formulated based on a re-interpretation of the defect-correction method. Very promising results have been obtained for second and higher order approximation, and with different types of stabilization.

We apply the designed mass-matrix-free residual distribution scheme for the mixed FEM/DG formulation of the Euler equations in Lagrangian reference frame and analyze its efficiency for realistic flow simulations.

Keywords: lagrangian hydrodynamics, residual distribution schemes, high-order finite element method, mass matrix

Mathematics Subject Classifications (2010): 65M, 76M

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