AN ITERATIVE DOMAIN DECOMPOSITION METHOD FOR EDDY CURRENT PROBLEMS ADAPTED TO THE GAUGE CONDITION

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Abstract. An iterative Domain Decomposition Method (DDM) is applied into a mixed formulation of eddy current problems with the gauge condition, which is an extension in case of magnetostatic problems; see Tagami [6].

From the engineering point of view, eddy current problems are often formulated by neglecting the gauge conditions, where the magnetic vector potential is only one unknown function; see, for example, Kameari–Koganezawa [2] and Kanayama, et al. [3]. The formulation without any gauge conditions enables us to reduce computational consts in case of the conventional one domain problem, and to formally introduce an iterative DDM. However, the formulation without any gauge conditions yields an indeterminate linear system. Therefore, to the best of our knowledge, mathematical justifications of numerical results such as unique solvability and convergency are not available and it is hard to generally apply direct solvers required for the reduction of computational costs of the iterative DDM. Moreover, when the formulation without any gauge conditions applied, iterative procedures diverge in case of large scale computational models whose numbers of degrees of freedom (DOF) are larger than 10^7; see, for example, Sugimoto, et al. [5].

To overcome difficulties mentioned above, a gauge condition is introduced and eddy current problems are formulated by mixed variational problems, which is related with the magnetostatic case in Kikuchi [4] and the eddy current case in Alonso–Valli [1]. The mixed formulation regards the magnetic vector potential and the Lagrange multiplier as two unknown functions that are approximated by the Nédélc curl-conforming edge element and by the conventional piecewise lienar element, respectively. Moreover, a reduced iterative procedure is established by means of the property of the Lagrange multiplier that vanishes in the whole domain.

Finally, some numerical results are shown in case of ultra-large computational models whose numbers of DOF are 10^7–10^9.

Keywords: domain decomposition method, mixed formulation, eddy current problem, gauge condition, finite element method

Mathematics Subject Classifications (2010): 65N55, 35M12, 83C50, 78M10

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