SOME APPLICATIONS OF THE LAGRANGE–GALERKIN METHOD IN FLOW PROBLEMS

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Abstract. A distinctive feature of fow problems is that the governing equations include the material derivative term

$$\frac{D\phi}{Dt} \equiv \frac{\partial \phi}{\partial t} + u \cdot \nabla \phi,$$

where u is a function expressing the flow field and ϕ is an unknown physical quantity such as the density, the velocity, or the energy. It makes the problems asymmetric and nonlinear when the velocity field is unknown, e.g., in the Navier-Stokes equations ϕ stands for each component u_i of unknown velocity u, which leads to the nonlinear term $u \cdot \nabla u_i$. The combination of this term with the diffusion term $-\nu\Delta\phi$ describes many important phenomena in sciences and engineering. It produces various fruitful and interesting results, especially when the diffusion constant ν is small, i.e., high Reynolds number problems in the Navier-Stokes equations. In devising numerical schemes for the solution of those phenomena, it is well-known that the discretization of this term is crucial because the conventional Galerkin finite element method and the centered difference method easily produce unphysical oscillating solutions. Among remedies for the instability the Lagrange-Galerkin method, which is also called characteristics finite element method [4] or Galerkin-characteristics method [1], is quite natural from the physical point of view since it approximates the particle movement along the trajectory. Thus, the method is not only robust for convection-dominated problems but also has the advantage that the resulting matrix is symmetric, which reduces much the computation cost.

In this talk we discuss some applications of the Lagrange-Galerkin method in flow problems on the following issues;

- pressure-stabilized scheme for the Navier-Stokes equations [2],
- stabilized scheme of second-order in time [3],
- energy-stable scheme for two-fluid flow problems [5].

Keywords: the finite element method, the Lagrange-Galerkin method, pressure-stabilization, the Navier-Stokes equations, two-fluid flow problems.

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