HIGH ORDER PENALTY METHODS: A FOURIER APPROACH TO SOLVING WAVE EQUATIONS ON DOMAINS WITH CURVED BOUNDARIES.

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Abstract. Penalty methods offer an attractive approach for solving partial differential equations (PDEs) on domains with curved or moving boundaries. In this approach, one does not enforce the PDE boundary conditions directly, but rather solves the PDE in a larger domain with a suitable source or penalty term. The new penalized PDE is then attractive to solve since one no longer needs to actively enforce the boundary conditions. Despite the simplicity, these methods have suffered from poor convergence rates which limit the accuracy of any numerical scheme (usually to first order at best). In this talk we will show how to systematically construct a new class of penalization terms which improve the convergence rates of the penalized PDE, thereby allowing for higher order numerical schemes. We will also show that the new penalized PDE has the added advantage of being solved in a straightforward manner using Fourier spectral methods. Finally, we demonstrate that the method is very general and works for elliptic (Poisson), parabolic (heat), and hyperbolic (wave) equations and will present time-domain simulations in two and three dimensions for complex geometries.

Keywords: extended domain, penalty methods, Fourier spectral methods

References

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