

# FINITE ELEMENT APPROXIMATION OF THE ISAACS EQUATION

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**ABSTRACT.** We propose and analyze a two-scale finite element method for the Isaacs equation. The fine scale is given by the mesh size  $h$  whereas the coarse scale  $\epsilon$  is dictated by an integro-differential approximation of the partial differential equation. We show that the method satisfies the discrete maximum principle, provided that the mesh is weakly acute. This, in conjunction with weak operator consistency of the finite element method, allows us to establish convergence of the numerical solution to the viscosity solution as  $h, \epsilon \rightarrow 0$  and  $\epsilon \geq Ch|\log h|$ . In addition, using a discrete Alexandrov Bakelman Pucci estimate and the known regularity results of the exact solution we deduce an algebraic rate of convergence for the method.

**Keywords:** Fully nonlinear equations, viscosity solution, discrete maximum principle.

**Mathematics Subject Classifications (2010):** 65N12, 65N15, 65N30, 35J60, 35D40, 35Q91.

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