# A PRIMAL-MIXED FINITE ELEMENT METHOD FOR THE ELECTROMECHANICS OF THE HEART

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ABSTRACT. In this talk we will present a finite element formulation for a coupled elasticity-reaction-diffusion system written in a fully Lagrangian form and governing the spatio-temporal interaction of species inside an hyper-elastic body (cf. [4]). Much of the classical work in the field of reaction-diffusion systems on moving domains focuses in specific instances (such as radial growth or uniaxial contraction in simple domains) where it is possible to characterize the body motion without resolving the underlying solid mechanics and yielding reaction terms written directly in terms of growth rate (see e.g. [2] and the references therein). Nevertheless here we are interested in problems where the type of movement is not known a priori and therefore both the deformation and the deformation gradients are needed to describe the kinematics of the system: roughly speaking, the displacements/deformations are employed to update the position of the domain, and their gradients appear explicitly in the diffusive terms and are required in the formulation of reaction-diffusion systems written in a deformed medium.

Here, a primal weak formulation is the baseline model for the reaction diffusion system written in the deformed domain, and a piecewise linear approximation will be employed. On the other hand, the strain is introduced as mixed variable in the elastodynamics, which in turn acts as coupling field needed to update the diffusion tensor of the modified reaction-diffusion system written in a deformed domain. The discrete setting consists of a mixed scheme based on row-wise Raviart-Thomas elements for stresses, Brezzi-Douglas-Marini elements for displacements, and piecewise constant pressure approximations.

Natural applications for the proposed framework are cardiac chemo- and electro-mechanical problems, however the present method is general enough to accommodate the study of many other related systems in biology and engineering. We close the presentation with some examples illustrating the accuracy and robustness of the proposed scheme.

**Keywords**: cardiac electromechanics, mixed finite elements, excitable media, moving domains, single cell mechanics, active strain.

**Mathematics Subject Classifications (2010)**: 65M60, 35K57, 35Q74.

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