MIXED-HYBRID FINITE ELEMENT METHODS FOR A NON-LINEAR DIFFUSION-REACTION EQUATION

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ABSTRACT. In many problems related to porous media applications it is needed to solve the nonlinear elliptic equation

$$\alpha(\mathbf{x},p)p - \operatorname{div}(K(\mathbf{x},p)\nabla p) = f, x \in \Omega \tag{1}$$

with appropriate boundary conditions, where Ω represents the porous medium domain, $p : \Omega \to \mathbb{R}$ represents a scalar variable of interest (such as concentration of a pollutant). Galerkin approximations for this problem typically employ globally continuous spaces for p. This approach guarantees good approximations for the scalar variable p, but in many cases the most important variable is the flux given by $\mathbf{u} = -K\nabla p$, what requires the definition of post-processing techniques [3]. An alternative way of numerically solving (1) is by writing the first order system

$$\alpha(\mathbf{x},p)p + \operatorname{div}\mathbf{u} = f, \qquad \mathbf{u} = -K(\mathbf{x},p)\nabla p \tag{2}$$

and using mixed formulations, where both the scalar and flux variables are simultaneously approximated. The main characteristic of mixed finite element methods is the use of different spaces for each variable, requiring compatibility conditions between these spaces [4]. In [1] the authors classify the different compatible spaces for the mixed problem in two families. One family has full $H(\mathrm{div})$-approximation where $\mathbf{u}$, p and div$\mathbf{u}$ are approximated to the same order, like the Raviart-Thomas (RT) elements of index $r \geq 0$, and the other family has reduced $H(\mathrm{div})$-approximation where p and div$\mathbf{u}$ are approximated to one less power, like the Brezzi-Douglas-Marini (BDM) elements of index $r \geq 1$ [4]. On general quadrilateral meshes these elements are defined on rectangles and extended to quadrilaterals using the Piola transform [2] what may create a consistency error and loss of approximation of the divergence of the flux. In this work we present a Picard-type mixed-hybrid finite element algorithm for the solution of (2) in quadrilateral meshes, using full and reduced $H(\mathrm{div})$ compatible spaces and exploring the different features of these approximations, such as accuracy, convergence and solution of the associated system of linear equations. Convergence studies are presented for problems where the analytical solution is known, to check the quality of the approximation of the scalar variable, the flux and its divergence on non-affine quadrilateral meshes.

Keywords: Finite Elements, Mixed-Hybrid Formulations, Nonlinear Elliptic Equation

Mathematics Subject Classifications (2010): Numerical Analysis, Partial differential equations

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