

hp-FEM FOR SINGULAR PERTURBATIONS: BALANCED NORMS AND MULTIPLE SCALES

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ABSTRACT. We consider the approximation of solutions of singularly perturbed elliptic equations. Two classes are studied. The first class consists of elliptic-elliptic problems. There, the classical Galerkin approximation is optimal in the so-called energy norm, which is, however, so weak that features within the layer are not well resolved. We show that robust exponential convergence of a high order FEM on suitable meshes is achieved also in a stronger, “balanced” norm. This latter norm is balanced in the sense that layer contributions are of size $O(1)$ uniformly in the perturbation parameter. Thus, the numerical approximation is also reliable in the layer. The second class consists of systems of singularly perturbed ODEs of elliptic-hyperbolic type with multiple scales. We discuss well-posedness of the variational formulations and robust convergence of an h -version Discontinuous Galerkin method on Shishkin meshes.

Keywords: *hp*-FEM, singular perturbation, balanced norms

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