ON NONLINEARLY AND LINEARLY IMPLICIT IMEX RUNGE-KUTTA METHODS FOR A CLASS OF DEGENERATE CONVECTION-DIFFUSION PROBLEMS.

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Abstract. Multi-species kinematic flow models with strongly degenerate diffusive corrections give rise to systems of nonlinear convection-diffusion equations of arbitrary size. Aplications to these systems include models of polydisperse sedimentation and multi-class traffic flow. Implicit-explicit (IMEX) Runge-Kutta (RK) methods [1] are suitable for the solution of these convection-diffusion problems since the stability restrictions, coming from the explicitly treated convective part, are much less severe than those that would be deduced from an explicit treatment of the diffusive term. These schemes usually combine an explicit Runge-Kutta scheme for the time integration of the convective part with a diagonally implicit one for the diffusive part. In [4], a nonlinear implicit IMEX-RK scheme of this type is proposed, where the nonlinear and non-smooth systems of algebraic equations arising in the implicit treatment of the degenerate diffusive part are solved by smoothing of the diffusion coefficients combined with a Newton-Raphson method with line search. A particularly efficient variant of these schemes, so-called linearly implicit IMEX-RK schemes [3], arise from discretizing the diffusion terms in a way that more carefully distinguishes between stiff and nonstiff dependence, such that in each time step only a linear system needs to be solved. In this talk a serie of examples of polydisperse sedimentation [2] and multi-class traffic flow [5] it is demonstrated that these linearly implicit IMEX-RK schemes approximate the same solutions as the nonlinearly implicit versions, and in many cases these schemes are more efficient.

Keywords: implicit-explicit Runge-Kutta schemes, degenerate convection-diffusion equations, linearly implicit methods, polydisperse sedimentation.

Mathematics Subject Classifications (2010): 65L06, 35L65, 76T20.

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