A HYBRID STOCHASTIC GALERKIN METHOD FOR UNCERTAINTY QUANTIFICATION APPLIED TO CLARIFER-THICKENER MODEL WITH SEVERAL RANDOM PERTURBATIONS

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Abstract. Many real world applications deals often with parameters which values are not known exactly. The uncertainty quantification techniques provide several methods to treat the uncertain problem parameters. The continuous sedimentation process in a clarifier-thickener can be modelled as a nonlinear conservation law, whose flux density function is discontinuous with respect to spatial position. In the application of this model the rate and the composition of the feed flow cannot be given deterministically.

We present the application hybrid stochastic Galerkin (HSG) method on the scalar clarifier-thickener model with several random perturbations. The HSG method extends the classical polynomial chaos approach by multi-resolution discretization in the stochastic space. The presented method improves the accuracy of the stochastic discretization and leads to a partially decoupled deterministic hyperbolic system, which allows the efficient parallel simulation. The further improvement of the computational efficiency is provided by the multi-wavelet based stochastic adaptivity. Several presented numerical experiments show the application of the HSG method on the scalar clarifier-thickener problem with several random parameters and demonstrate advantages of the method.

Keywords: clarifier-thickener model; polynomial chaos; uncertainty quantification; multiple stochastic dimensions; hybrid stochastic Galerkin; finite volume method.

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