SPACE-TIME TREFFTZ DISCONTINUOUS GALERKIN METHODS FOR WAVE PROBLEMS

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ABSTRACT. A space-time discontinuous Galerkin method for wave propagation problems will be presented and analysed, following [1]. The special feature of the scheme is that it is a Trefftz method, namely that trial and test functions are solution of the partial differential equation to be discretised in each element of the (space-time) mesh. The method considered is a modification of the discontinuous Galerkin schemes of [2] and of [3].

For acoustic and electromagnetic wave equations in one space dimension, we prove stability of the method, quasi-optimality, best approximation estimates for polynomial Trefftz spaces, and high order error bounds in the meshwidth and in the polynomial degree. The analysis framework also applies to the case of higher space dimensions. Some numerical experiments demonstrate the theoretical results and the faster convergence compared to the non-Trefftz version of the scheme.

Keywords: Discontinuous Galerkin method, Trefftz method, space–time finite elements, wave propagation, Maxwell equations, a priori error analysis, approximation estimates

Mathematics Subject Classifications (2010): 65M12, 65M60, 35L05

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